Exercise 37

(a) If $f(x) = (x^3 - x)e^x$, find $f'(x)$.

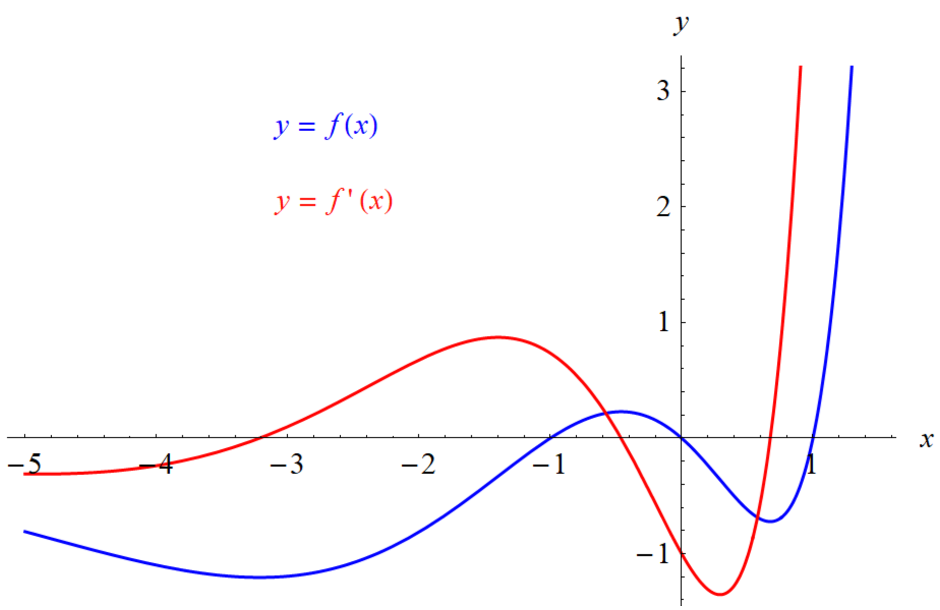
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

Solution

Evaluate the derivative using the product rule.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[(x^3 - x)e^x\right] \\ &= \left[\frac{d}{dx}(x^3 - x)\right](e^x) + (x^3 - x)\left[\frac{d}{dx}(e^x)\right] \\ &= (3x^2 - 1)(e^x) + (x^3 - x)(e^x) \\ &= (x^3 + 3x^2 - x - 1)e^x \end{aligned}$$

Below is a graph of the function and its derivative versus x.



$f'(x)$ is positive wherever $f(x)$ increases, $f'(x)$ is zero wherever the slope of $f(x)$ is zero, and $f'(x)$ is negative wherever $f(x)$ is decreasing. The answer to part (a) is reasonable then.